Immersive visualization of data sets in the 3-sphere

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Summary

For this project, we developed real-time 3D graphics programs for visualisation of point-cloud data sets in the 3-dimensional sphere. They target both virtual reality platforms (such as Google Cardboard and Oculus Rift) and desktop computers. We used these programs to explore data sets related to the geometry and combinatorics of curves on compact hyperbolic surfaces.

Our faculty supervisor was David Dumas and our graduate mentors were Nathan Lopez and Jasmine Otto.

The Problem

The 3-sphere is a higher dimension analogue of a sphere. It lives in the 4-dimensional space. Since it is difficult to visualise the 4-dimensional space, we use stereographic projection (i.e. a mapping function that projects a sphere onto a plane) to picture this in a 3-dimensional space. This type of projection does not preserve the distances nor the volume but it does preserve the angles. The two type of data sets that we use are:

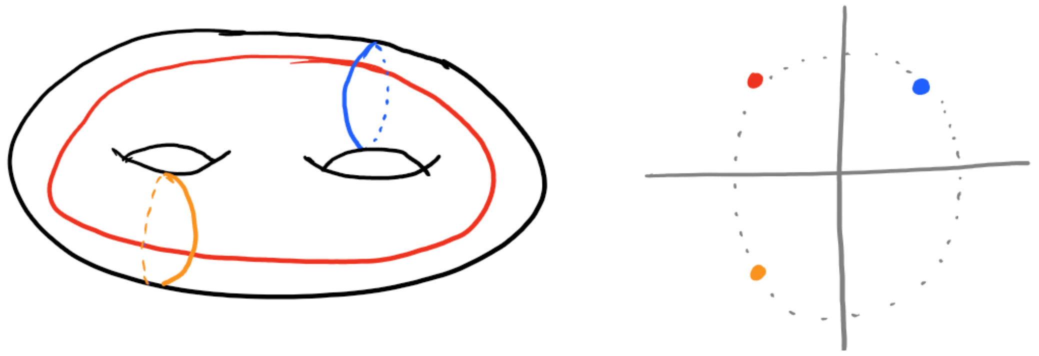
1. Lattices in $\mathbb{R}^4$ such as the integer lattice $\mathbb{Z}^4$
2. Simple closed curves on hyperbolic surfaces [1]

Implementation

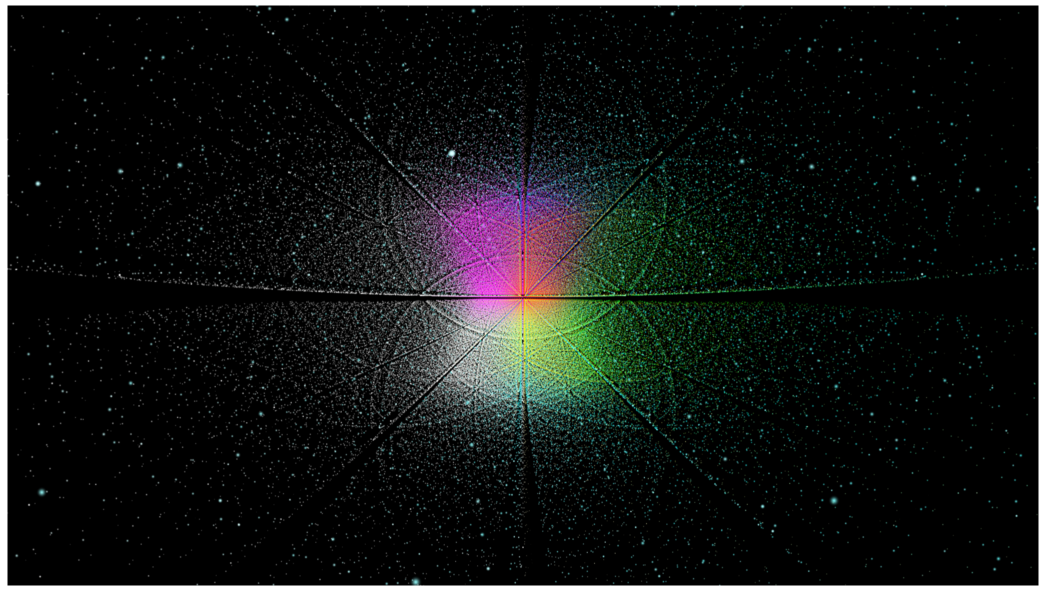
To visualise the data set in a useful way, we created two programs. The first is built using Unity 3D graphics framework and the other is a browser-based WebGL application. Each program uses a point cloud to visualise a particular dataset. Each point is visible from a first-person perspective. This gives the user the ability to fly around the points. The Unity-based program has partial support for point clouds tagged with additional features, which can be depicted in brightness, colour, or size of the displayed point. For example, categories of points become colours and weights on points become size or brightness. WebGL, because of its limitations, makes it harder to implement the points directly from a particular data set. To make it easier, a third party library had to be used to implement the points [2].

Point clouds from curves

Below is a picture of a genus-2 surface with three simple closed curves. Next to it are the same curves but as points in $\mathbb{R}^n$. A more general version of this example would be, using the work of W. Thurston, the set of simple closed curves on a hyperbolic surface is realised as a dense set of points in a topological n-sphere, where n depends on the surface [3].

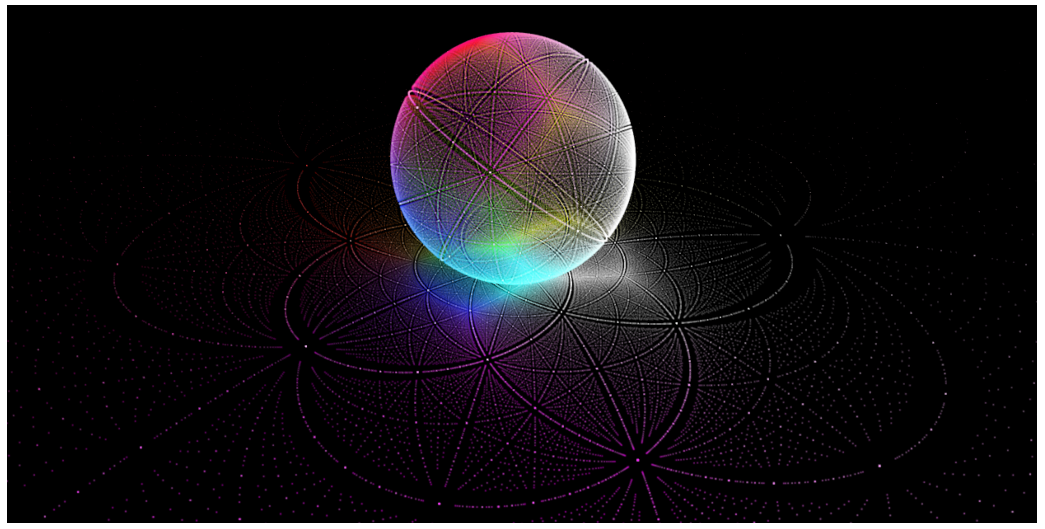


Screenshots (4D Lattice)



These are the primitive integer vectors in $\mathbb{R}^4$, normalized and seen in stereographic projection using our program.

Screenshots (3D Lattice)



For comparison, these are the primitive integer vectors in $\mathbb{R}^3$. Since this is a dimension lower, the projection process and the actual sphere can be visualized.

Future Directions

The next step would be to allow the user to select a point in the simple curve data set and have a representative of the curve show up in a heads-up display. We would also like to port the program to other platforms and to use it for other 4-dimensional lattices.

References

[1] D. Dumas and F. Guéritaud, The PML Visualization Project. September 2015. http://dumas.io/PML/

[2] Schutz, M. Potree. http://potree.org. Accessed: 2015-11-28

[3] W. Thurston, Minimal stretch maps between hyperbolic surfaces. Preprint, 1986. arXiv:math/9801039