Wikipedia Illustration Task Force

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Summary

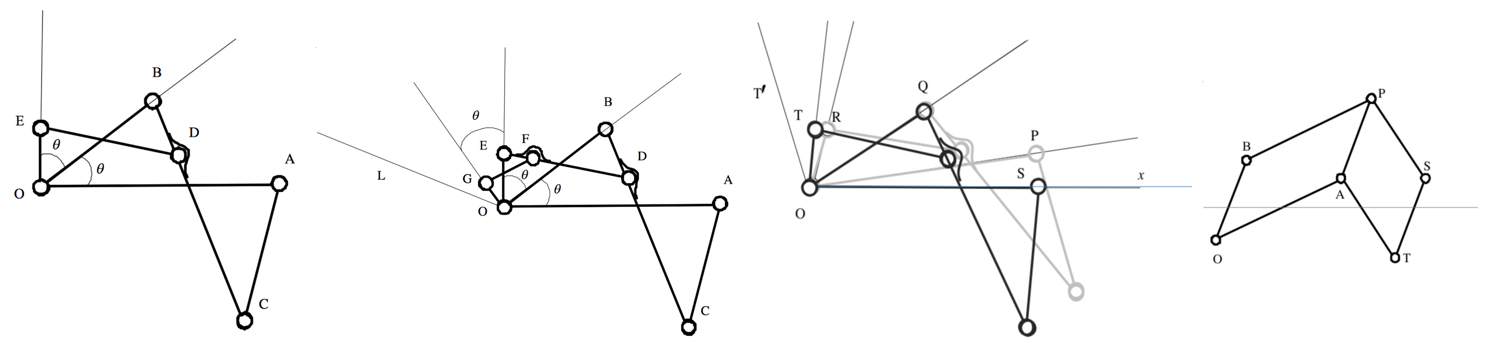
The purpose of this project was to use programming techniques and tools to create mathematical illustrations (animations, graphs, diagrams) that could enrich Wikipedia articles. The main focus was on algebraic curves and how they may be drawn using Kempe's Universality Theorem.

Kempe's Universality Theorem

For an arbitrary algebraic plane curve a linkage can be constructed that draws the curve.

Kempe's Mechanisms

- Four calculating linkages: reversor, multiplicator, additor, translator



Below is an example of Kempe's mechanisms consisting of 48 links and 70 joints. It moves point P to draw two intersecting lines given by the equation

$$(x-y)(x+y+\frac{1}{\sqrt{2}})=0$$

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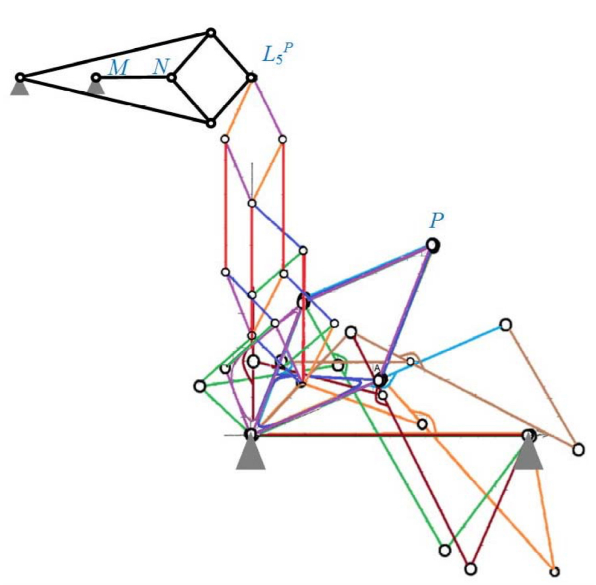


- A generalization of Kempe's theorem tells us that a curve of degree n requires at least $\mathcal{O}(n^2)$ bars

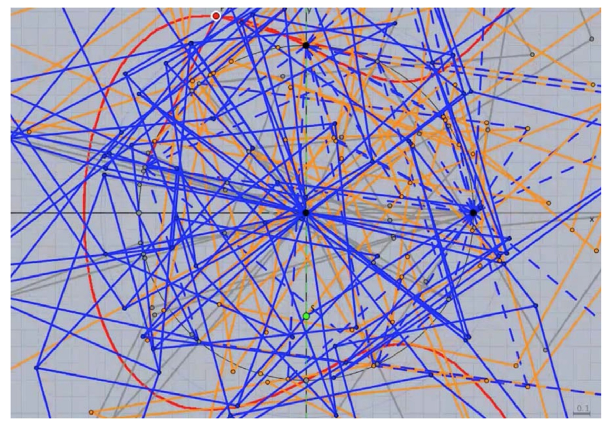
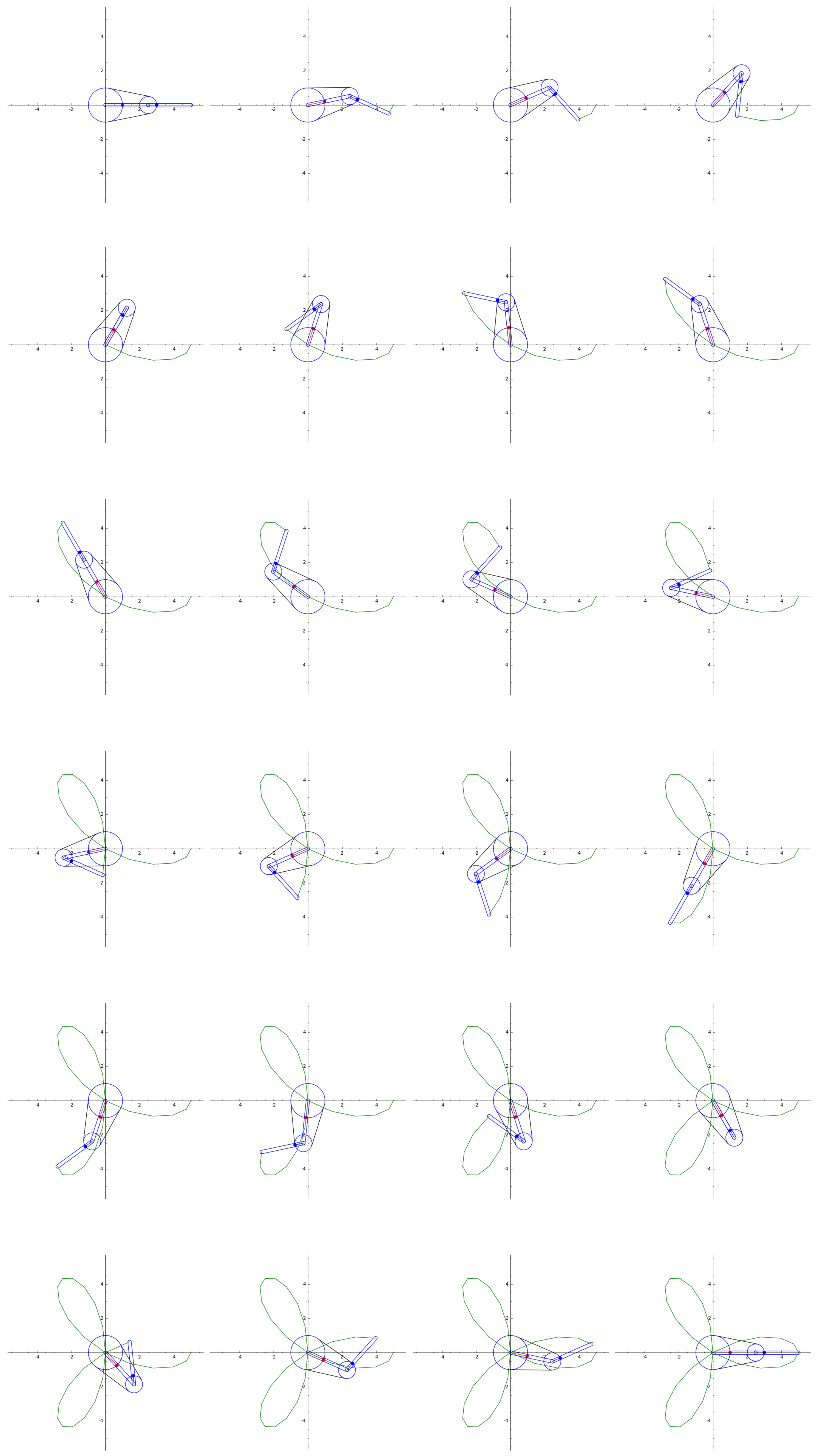


Figure: The dynamic geometry system Cinderella shows the linkages that would draw an elliptic cubic curve (the curve can be seen in red).

- Kempe himself asked for a mathematical artist to discover the simplest linkworks that will describe particular curves

Animation



Trigonometric Plane Curves

A trigonometric plane curve, is a parameterized curve with coordinate functions that are finite Fourier series, shown as the equation:

$$P=\begin{Bmatrix} x(\theta) \\ y(\theta) \end{Bmatrix} = \begin{Bmatrix} \sum_{k=0}^{m} a_k \cos k\theta + b_k \sin k\theta \\ \sum_{k=0}^{m} c_k \cos k\theta + d_k \sin k\theta \end{Bmatrix}$$

where a_k, b_k, c_k, and d_k, $k=0,\ldots,m$, are the real coefficients and $\theta \in [0,2\pi]$.

To create the animation in the column to the left, we used the trigonometric plane curve equation for the trifolium:

$$P_T=\begin{cases} -\cos 2\theta - \cos 4\theta \\ \sin 2\theta - \sin 4\theta \end{cases}$$

Sage

In order to create the single coupled serial chain that draws our curves, we had to break down its components: two circles, a crank, two chains, two arms, and the curves being drawn. For every frame, our program calculated the position of these components connected to the same input angle θ and driven by chains at increasing speeds by pulleys of decreasing radius.

Conclusions

The goal of this project was to provide illustrations to mathematical Wikipedia pages that could be better understood through visual applications of its concept. We were able to delve into Kempe's Universality Theorem and understand it through sources used in the article to recreate the mechanisms that draw algebraic curves. Using Sage, we animated a mechanism to draw both the trifolium and hypocycloid curves and post gifs of it to the Wikipedia article to aid future readers.

References

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