3D Printing Surfaces

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Summary

The research project explored surfaces in three-dimensional space by 3D printing models of them. The goal was to start by choosing appropriate equations and end with physical models which would be appropriate for use in multi-variable calculus classes. It involved learning about function graphing software, 3D modeling software and 3D printing. The results varied from exploring equations from multi-variable calculus to ones based on each others curiosity.

Motivation

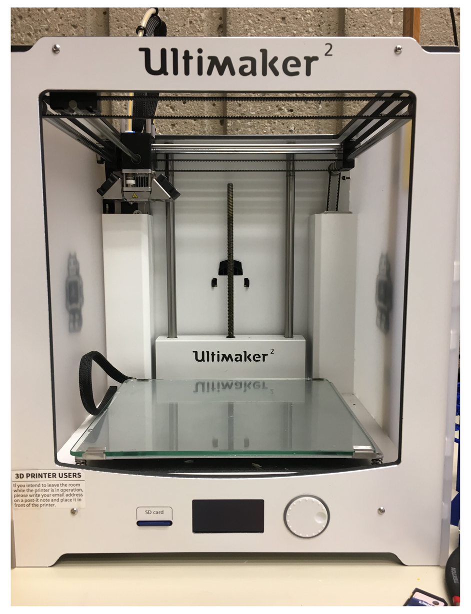
Our Motivation was to satisfy our curiosity about how equations from multi-variable calculus can become 3D objects since we learned such equations on 2D paper. Accomplishing this was satisfying to see and hold such physical models such as Hyperboloid of One Sheet, Paraboloid, and a lovely shape as well as others.

Methodology

The methodology consisted of trial and error of simple shapes/equations that were common and small scale. Smaller objects that were printed at first that helped us identify problems and other mistakes that could possibly affect our prints in later stages. As time passed, we began to increase the difficulty by choosing more obscure equations that lead to quadric surfaces from multi-variable calculus and other uncommon equations.

Equipment/Materials

3D printer: Ultimaker 2 from Ultimaker



Material that models are made:

- PLA plastic (Polylactic acid)

Lists of Shapes Printed

- Hyperboloid of One Sheet
- Pentagonal Hyperboloid of One Sheet
- Twisted Pentagonal Hyperboloid of One Sheet
- Cosinus
- Paraboloid
- Heart
- Hexahedron
- Gear cog

The Math

Twisted Pentagonal Hyperboloid of One Sheet:

$$(\sigma,\theta,z)r \leq f(z)\left(\frac{\left(\frac{\cos(\theta)-g(z)}{5}\right)}{\cos\left(\left(\left(\frac{\theta-g(z)}{5}\right) \mod \frac{2\pi}{5}\right)-\frac{\pi}{5}\right)}\right)$$

Parametric equation:

$$x : a*\sqrt{1+u^2}*\cos(v)$$
$$y : b*\sqrt{1+u^2}*\sin(v)$$
$$z : c*u$$
$$u_{max} : \pi, u_{min} : -\pi, v_{max} : \pi, v_{min} : -\pi, v_{step} : 5, twist : 360^\circ$$

Cosinus:

$$\left\{x = u, y = sin(\pi((u)^2+(v)^2))/2,, z = v \quad t \in (-\infty,\infty)\right.$$

Hexaedron:

$$\left\{x = cos(v)^3 * cos(u)^3, y = sin(u)^3,, z = sin(v)^3 * cos(u)^3\right.$$
$$u \in (-\pi/2, \pi/2), v \in (2\pi)$$

Early Attempts

At first things were mostly like this...

Printed Results

Twisted Pentagonal Hyperboloid of One Sheet:

Cosinus:

Hexaedron:

Gear Cog:

Software Used

The software used from modeling to export .obj or. stl files to the software Cura, that comes natively with Ultimaker, to print 3D models.

- Blender
- SageMath
- Wolfram Mathematica

Software Obstacles

Use of software was heavily used during the duration of this research. The use of Blender, SageMath, Wolfram Mathematica, and Cura. Each software proved its own difficulty during the process of creating, modeling, and especially printing each of our models. In Blender we initially found modeling, scaling and setting the thickness of models confusing. SageMath was difficult because, without the use of correct syntax, certain models could not be created. Using Wolfram Mathematica proved to be difficult because trying to export models was not always consistent. Cura was relatively simple and straightforward, except adding support structures and percentages of "infill" were not easy to understand at first.

Conclusion

- Helped students visualize 2D surfaces into 3D models
- Learned about function graphing software, 3D modeling software, and 3D printing
- Explored surfaces in three-dimensional space by 3D printing models of them
- Turning equations into 3D objects is more difficult than it seems.

We would also like to thank Daniel Groves and Paul Rapoport for helping and supporting us all during the duration of the project. Without their guidance and expertise in the field, a lot of the results would not have been possible.

References/Resources

To get a pentagon on a plane:
https://math.stackexchange.com/questions/41940/is-there-an-equation-to-describe-regular-polygons (From user: Raskolnikov)
Blender Documents:
https://wiki.blender.org/index.php/
Ideas:
https://www.utdallas.edu/ jwz120030/3DPrintedModelsForCalcIII/